

PRECISE POSITIONING OF A TRANSPONDER PLACED ON A SEA FLOOR (2)

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Abstract: In the previous paper, it is shown that not only the position of a bottom transponder but also the underwater acoustic velocity distribution can be determined, if the position of surface transponders and ranges between the surface and bottom transponders are known. In the present paper, the theory is extended to include the effects of current. The basic nonlinear equations are obtained by a variational principle. Some two-dimensional numerical calculations are conducted, and the convergence of the iteration calculations is verified. The numerical results show that the effects of current can be obtained by the present method and can't be neglected for precise positioning.

1 INTRODUCTION

In the first report and Ref [2], if the position of sea surface transponders and the distance between the surface and bottom transponders are given, not only the position of a sea bottom transponder but also the underwater acoustic velocity can be determined. A division into two or three horizontal layers seems to be sufficient for the estimation of the correct positioning of the bottom transponder.

In Ref. [3], a theoretical possibility of obtaining the position of the sea bottom transponder with accuracy of centimeter order by using measuring instruments available at present is shown. For the precise estimation of the bottom transponder position, the simultaneous estimation of the underwater acoustic velocity distribution and the position of the underwater transponder is shown to be extremely important.

In these papers, two-dimensional calculations are conducted. In Ref [4], more realistic simulations are conducted by three-dimensional calculations. And a new idea called MIL (Method of Incremental Layers) is introduced to stabilize numerical calculations. Specifically, the number of layers is increased one by one. The initial value of the iteration calculation is obtained from the previous layer division. It is confirmed that MIL improves the stability of convergence calculation.

In the previous reports, the current velocity was taken zero. In the present paper, not only the effect of the underwater acoustic velocity distribution but also those of the current velocity distribution is discussed. The basic equations are obtained, and some numerical

calculations are also conducted.

2. BASIC EQUATIONS

Let x and y be horizontal axes and z vertical axis as shown in Fig. 1. The underwater acoustic velocity and current are supposed to vary as $C(z)$ and $(U_x(z), U_y(z), 0)$ for $-h \leq z \leq 0$, where h is water depth. The acoustic ray $x = x(z), y = y(z), (z_a \geq z \geq z_b)$ connecting points $P_a(x_a, y_a, z_a)$ and $P_b(x_b, y_b, z_b)$ is solution of a variational problem⁴⁾:

$$\begin{aligned} t[x, y] &= \int_{z_a}^{z_b} \frac{ds}{C(z) + [U_x(dx/ds) + U_y(dy/ds)]} \\ &= - \int_{z_a}^{z_b} \left[(dx/dz)^2 + (dy/dz)^2 + 1 \right] dz / [C(z) \\ &\quad \sqrt{(dx/dz)^2 + (dy/dz)^2 + 1} - U_x(dx/dz) - U_y(dy/dz)] \\ &= \min_{\text{under}} \end{aligned} \quad (1a)$$

$$\begin{aligned} x(z_a) &= x_a = \text{given}, & y(z_a) &= y_a = \text{given}, \\ x(z_b) &= x_b = \text{given}, & y(z_b) &= y_b = \text{given} \end{aligned} \quad (1b)$$

where the functional $t[x, y]$ is time necessary for the acoustic wave to travel from P_a to P_b , and ds is $\sqrt{dx^2 + dy^2 + dz^2}$.

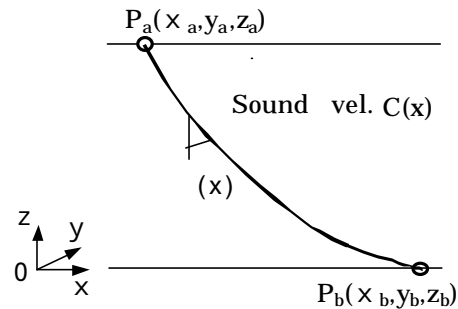


Fig. 1 Sound propagating underwater

The Euler equations of the variational problem (1) give

$$\begin{aligned} &[-C(z)\sin q_x(z) + U_x(z)\cos 2q_x(z) \\ &\quad - 2U_y(z)\sin q_x(z)\sin q_y(z)] / \end{aligned}$$

$$\begin{aligned} & [C(z) + U_x(z) \sin \mathbf{q}_x(z) + U_y(z) \sin \mathbf{q}_y(z)]^2 \\ & = \text{const} \end{aligned} \quad (2a)$$

$$\begin{aligned} & [-C(z) \sin \mathbf{q}_y(z) + U_y(z) \cos 2\mathbf{q}_y(z) \\ & - 2U_x(z) \sin \mathbf{q}_x(z) \sin \mathbf{q}_y(z)] / \\ & [C(z) + U_x(z) \sin \mathbf{q}_x(z) + U_y(z) \sin \mathbf{q}_y(z)]^2 \\ & = \text{const} \end{aligned} \quad (2b)$$

where

$$\sin \mathbf{q}_x(z) = dx/ds \quad (3a)$$

$$\sin \mathbf{q}_y(z) = dy/ds \quad (3b)$$

Eq. (3) is nothing but Snell's law. If angle between vectors ds and dz is denoted by I and angle

between vectors $ds - dz$ and dx by m , relation between $\mathbf{q}_x, \mathbf{q}_y$ and I, m is given as

$$dx/ds = \sin \mathbf{q}_x(z) = \sin I \cos m \quad (4a)$$

$$dy/ds = \sin \mathbf{q}_y(z) = \sin I \sin m \quad (4b)$$

Various problems are born depending on whether the acoustic velocity C , current velocity U , inclination angle \mathbf{q} of acoustic rays and bottom transponder position x_B are considered known or unknown. The problems are classified in Table 1.

Table 1 Classification of problems

No.	Case Name	Acoustic Velocity c_m	Current Velocity U_m	Inclination Angle \mathbf{q}_{mm}	Position of Transponder x_B
1	SUTP1101	given	given	unknown	Given
2	SUTP1100	given	given	unknown	unknown
3	SUTP1000	given	unknown	unknown	unknown
4	SUTP0100	unknown	given	unknown	unknown
5	SUTP0000	unknown	unknown	unknown	unknown

3. A NUMERICAL PROCEDURE FOR TWO-DIMENSIONAL PROBLEMS

In the following, slowness $S(z)$ is used instead of sound velocity $C(z)$:

$$S(z) = 1/C(z) \quad (5)$$

As shown in Fig. 2, the acoustic field is divided into horizontal layers to discretize the problem. The discretized equations are shown below. In the following equations, m and n refer to ray and layer.

Snell's Law:

$$\begin{aligned} & \frac{S_n (\sin \mathbf{q}_{mn} - S_n U_n \cos 2\mathbf{q}_{mn})}{(1 + S_n U_n \sin \mathbf{q}_{mn})^2} \\ & = \frac{S_{n+1} (\sin \mathbf{q}_{m,n+1} - S_{n+1} U_{n+1} \cos 2\mathbf{q}_{m,n+1})}{(1 + S_{n+1} U_{n+1} \sin \mathbf{q}_{m,n+1})^2} \\ & (m = 0, 1, \dots, M-1; n = 1, 2, \dots, N-1) \end{aligned} \quad (6)$$

Travel time of sound signal:

$$\begin{aligned} \Delta t_m &= \sum_{n=0}^{N-1} \Delta t_{mn} = \sum_{n=0}^{N-1} \frac{h_n}{\cos \mathbf{q}_{mn} C_n - U_n \sin \mathbf{q}_{mn}} \\ &= \sum_{n=0}^{N-1} \frac{h_n S_n}{\cos \mathbf{q}_{mn} (1 - S_n U_n \sin \mathbf{q}_{mn})}, \\ & (m = 0, 1, \dots, M-1) \end{aligned} \quad (7)$$

Horizontal distance between the surface and bottom transponders:

$$x_B - x_{Fm} = \sum_{n=0}^{N-1} h_n \tan \mathbf{q}_{mn}, \quad (m = 0, 1, \dots, M-1) \quad (8)$$

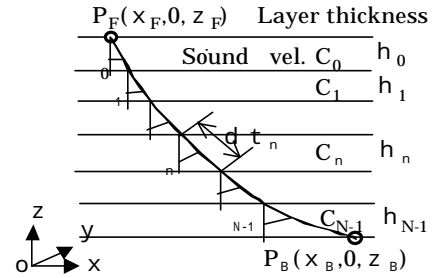


Fig. 2 Approximation of sound field

The Eqs. (6), (7) and (8) form a closed system. Namely, the number of unknowns $\mathbf{q}_{mn}, C_n, U_n, x_B$; $m = 0, \dots, M-1, n = 0, \dots, N-1$ is $MN + 2N + 1$. On the other hand, the number of equations is $M(N-1) + 2M$. Hence, if

$$M = 2N + 1, \quad (9)$$

the number of the unknowns coincides with that of the equation.

Snell's Law given by Eq. (6) can be rewritten as

$$\frac{S_n (\sin \mathbf{q}_{mn} - S_n U_n(z) \cos 2\mathbf{q}_{mn}(z))}{(1 + S_n U_n \sin \mathbf{q}_{mn})^2} = k_m \quad (10)$$

Hence, the unknowns \mathbf{q}_{mn} can be replaced by the new unknowns k_m . An algorithm to eliminate $\mathbf{q}_{mn}(z)$ is given as follows.

Let S_n, U_n and k_m be given. Then, \mathbf{q}_{mn} is obtained by solving Eq. (10). Since Eq. (10) can't be

solved analytically, a numerical solution is obtained by using Newton-Raphson method:

$$\begin{aligned} & \frac{S_n (\sin \mathbf{q}_{mn} - S_n U_n(z) \cos 2\mathbf{q}_{mn}(z))}{(1 + S_n U_n \sin \mathbf{q}_{mn})^2} \\ & + \left\{ \frac{S_n [\cos \mathbf{q}_{mn} + 2S_n U_n \sin 2\mathbf{q}_{mn}]}{[1 + S_n U_n \sin \mathbf{q}_{mn}]^2} \right. \\ & \quad - 2S_n [\sin \mathbf{q}_{mn} - S_n U_n \cos 2\mathbf{q}_{mn}] \cdot \\ & \quad \left. \frac{S_n U_n \cos \mathbf{q}_{mn}}{[1 + S_n U_n \sin \mathbf{q}_{mn}]^4} \right\} d\mathbf{q}_{mn} = k_m \end{aligned} \quad (11)$$

The dependence between \mathbf{q}_{mn} , S_n , U_n and k_m are obtained by differentiating Eq. (12) as

$$\begin{aligned} & \left\{ \frac{S_n [\cos \mathbf{q}_{mn} + 2S_n U_n \sin 2\mathbf{q}_{mn}]}{[1 + S_n U_n \sin \mathbf{q}_{mn}]^2} \right. \\ & \quad - 2S_n [\sin \mathbf{q}_{mn} - S_n U_n \cos 2\mathbf{q}_{mn}] \cdot \\ & \quad \left. \frac{S_n U_n \cos \mathbf{q}_{mn}}{[1 + S_n U_n \sin \mathbf{q}_{mn}]^3} \right\} d\mathbf{q}_{mn} \\ & - \left\{ \frac{\sin \mathbf{q}_{mn}}{[1 + S_n U_n \sin \mathbf{q}_{mn}]^2} \right. \\ & \quad \left. - \frac{2[\sin \mathbf{q}_{mn} - S_n U_n \cos 2\mathbf{q}_{mn}]}{[1 + S_n U_n \sin \mathbf{q}_{mn}]^3} \right\} dS_n \\ & + \left\{ -\frac{S_n^2 \cos 2\mathbf{q}_{mn}}{[1 + S_n U_n \sin \mathbf{q}_{mn}]^2} \right. \\ & \quad - 2S_n [\sin \mathbf{q}_{mn} - S_n U_n \cos 2\mathbf{q}_{mn}] \cdot \\ & \quad \left. \frac{S_n \sin \mathbf{q}_{mn}}{[1 + S_n U_n \sin \mathbf{q}_{mn}]^3} \right\} dU_n \\ & = dk_m \end{aligned} \quad (12)$$

Eqs. (10) and (12) can be used to eliminate \mathbf{q}_{mn} in Eqs. (13) and (14).

The nonlinear equations (7) through (8) are solved by Newton-Raphson Method as follows.

Travel time of acoustic signal:

$$\begin{aligned} \Delta t_m &= \sum_{n=1}^N \frac{h_n}{\cos \mathbf{q}_{mn}} \frac{S_n}{1 - S_n U_n \sin \mathbf{q}_{mn}} \\ & + \sum_{n=1}^N \left\{ \left[\frac{h_n}{\cos \mathbf{q}_{mn}} \frac{1}{1 - S_n U_n \sin \mathbf{q}_{mn}} \right. \right. \\ & \quad \left. \left. + \frac{h_n}{\cos \mathbf{q}_{mn}} \frac{S_n U_n \sin \mathbf{q}_{mn}}{(1 - S_n U_n \sin \mathbf{q}_{mn})^2} \right] dS_n \right. \\ & \quad \left. + \left[\frac{h_n \sin \mathbf{q}_{mn}}{\cos^2 \mathbf{q}_{mn}} \frac{S_n}{1 - S_n U_n \sin \mathbf{q}_{mn}} \right] dU_n \right\} \end{aligned}$$

$$\begin{aligned} & + \frac{h_n}{\cos \mathbf{q}_{mn}} \frac{S_n^2 U_n \cos \mathbf{q}_{mn}}{(1 - S_n U_n \sin \mathbf{q}_{mn})^2} \Big] d\mathbf{q}_{mn} \\ & + \left[\frac{h_n}{\cos \mathbf{q}_{mn}} \frac{S_n^2 \sin \mathbf{q}_{mn}}{(1 - S_n U_n \sin \mathbf{q}_{mn})^2} \right] dU_n \Big\}, \\ & (m = 0, 1, \dots, M-1) \end{aligned} \quad (13)$$

Horizontal distance between the surface and bottom transponders:

$$\begin{aligned} x_B - x_{Fm} + dx_B &= \sum_{n=1}^N h_n \tan \mathbf{q}_{mn} + \sum_{n=1}^N \frac{h_n}{\cos^2 \mathbf{q}_{mn}} d\mathbf{q}_{mn}, \\ & (m = 0, 1, \dots, M-1) \end{aligned} \quad (14)$$

4. NUMERICAL RESULTS IN TWO-DIMENSIONAL PROBLEMS

Distribution of underwater sound and current velocity used in numerical calculations are shown in Fig. 3. The water depth is 1000m, and the region is divided into 40 layers.

In the following Eqs. (10) through (14) are used. First, Problem 2 in Table 1 was solved. In this problem, the sound and current velocities are assumed given. The number of unknowns $k_m, x_B; m = 1, \dots, M, n = 1, \dots, N$ is $M+1$, and that of equations is $2M$. Hence, if

$$M = 1, \quad (15)$$

the number of the unknowns coincides that of the equations. When the number of the acoustic rays are bigger than one, the least square procedure is applied.

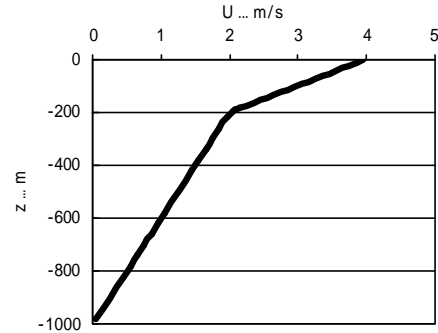


Fig. 3a Distribution of acoustic velocity C

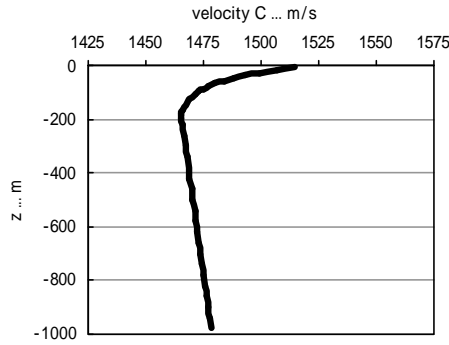


Fig. 3b Distribution of current velocity U

In the following, five rays are used. The characteristics of the five rays are given in Table 2. The x -coordinate of the bottom transponder x_B is assumed zero.

Table 2 Characteristics of sound rays

	Ray 0	Ray 1	Ray 2	Ray 3	Ray 4
Pos. of Surface Transponder	-448.880m	-669.563	-943.529	-1311.899	-1854.256
One Way Travel Time	0.744288s	0.817281	0.933793	1.120475	1.431079
Incidence Angle of Ray*	25deg	35	45	55	65
Average	24.174	33.804	43.334	52.679	61.650

The angle between the ray and the negative z direction at the surface.

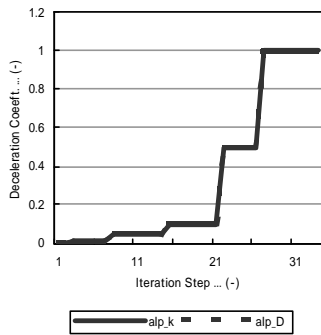


Fig 4a Selection of deceleration coefficients a (SUTP1100; 40 layers)

Fig. 4 shows the results for a case where the underwater field is divided into 40 layers. a s are deceleration coefficients in iteration procedure²⁾⁻⁴⁾. As shown in Fig. 4a, a for k_m (i.e. alp_k) and that for x_B (i.e. alp_D) should be different in general. The convergence of the iteration procedure was very nice in spite of the big number of the layer division. As can be seen from Fig. 4e, the effect of the current velocity on the ray trajectory seems not significant, but the effect can't be neglected for the precise positioning as premised in the present paper as shown in Fig. 4f.

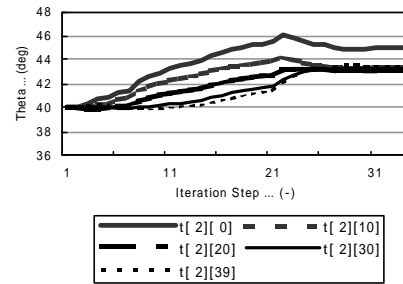


Fig. 4b Convergence of ray inclination angles q (SUTP1100; 40 layers; $t[m][n]$ denotes q_{mn})

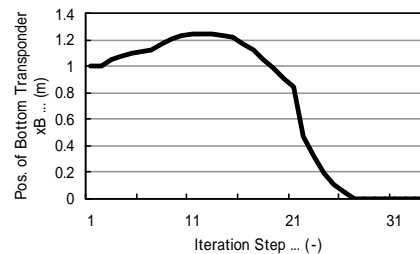


Fig. 4c Convergence of horizontal position of bottom transponder x_B (SUTP1100; 40 layers)

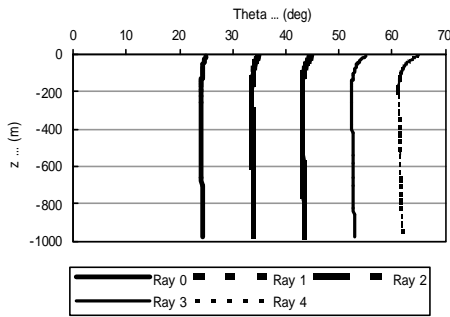


Fig. 4d Convergence of ray inclination angles q (SUTP1100; 40 layers)

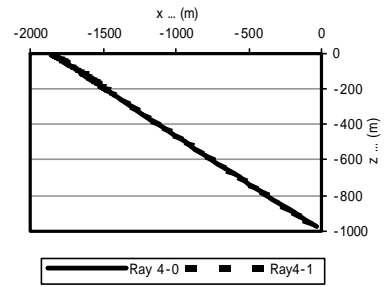


Fig. 4e Effect of current velocity on ray trajectory (SUTP1100; 40 layers; Ray 4-0 and Ray-1 denote with and without current)

Table 3 A comparison of the acoustic rays and the horizontal position of the bottom transponder between 40 layers and 1 layer divisions (SUTP1100)

		Ray 0	Ray 1	Ray 2	Ray 3	Ray 4	x_B
40 layers	Incidence	25deg	35	45	55	65	0m
	Average	24.174	33.804	43.334	52.679	61.650	
1 layer		24.175	33.805	43.336	52.684	61.662	0.007

Table 4 The sound velocity, the current velocity, the rays and the horizontal position of the bottom transponder (SUTP0000; 1 layer)

	C	U	Ray 0	Ray 1	Ray 2	Ray 3	Ray 4	x_B
40 layer Average	1473.33 m/s	1.4 m/s	24.174 deg	33.804	43.334	52.679	61.650	0m
1 layer	1473.3	1.3	24.183	33.812	43.341	52.687	61.664	0.17

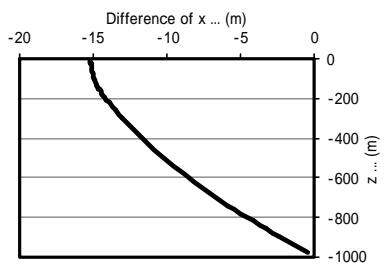


Fig. 4f Difference between ray trajectories with and without current (SUTP1100; 40 layers; Ray 4)

Fig. 5 shows the results for a case where the underwater field is divided into 1 layer. In the calculation, the acoustic and current velocities are the average of 40 layers, that is, $\bar{C} = 1473.33 \text{ m/s}$ and $\bar{U} = 1.4 \text{ m/s}$. A comparison of the acoustic rays and the horizontal position of the bottom transponder between 40 layers and 1 layer divisions is made in Table 3.

The one layer solution gives a very nice approximation unexpectedly. The average slopes of rays in case of 40 layers division agree very well with those in case of 1 layer division. The horizontal position of the bottom transponder x_B in case of 1 layer division is 0.007m, where the correct value is 0m.

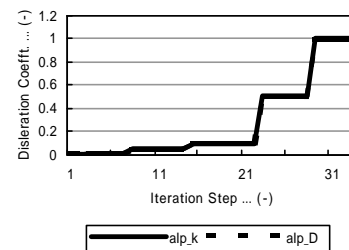


Fig 5a Selection of deceleration coefficients a (SUTP1100; 1 layer)

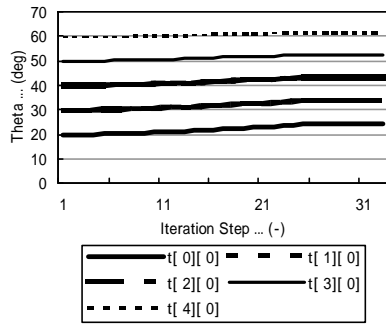


Fig. 5b Convergence of ray inclination angles q (SUTP1100; 1 layer; $t[m][n]$ denotes q_{mn})

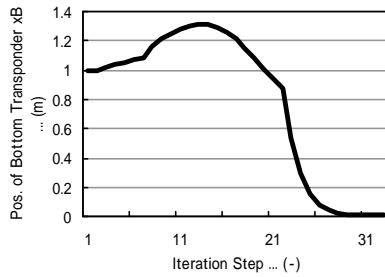


Fig. 5c Convergence of horizontal position of bottom transponder x_B (SUTP1100; 1 layer)

Since the sound and current velocities may be unknown in many cases, the results in these situations are shown in Fig. 6. In Fig. 6a, alp_S , alp_k , alp_U and alp_D denote the deceleration coefficients for S_m , U_m , q_{mn} and x_B . The sound velocity, the current velocity, the rays and the horizontal position of the bottom transponder are summarized in Table 4. In this calculation, the layer division is one. The results seem to be reasonable, although the precision of x_B is not sufficient. To increase the accuracy, the number of the layers must be increased.

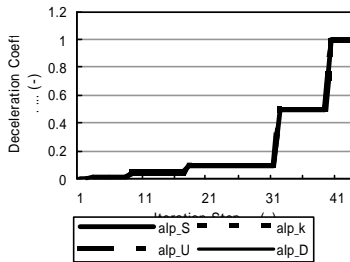


Fig 6a Selection of deceleration coefficients a (SUTP0000; 1 layer)

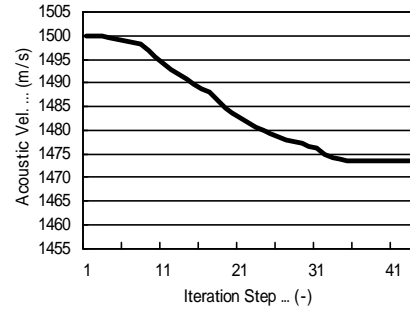


Fig. 6b Convergence of sound velocity C (SUTP0000; 1 layer)

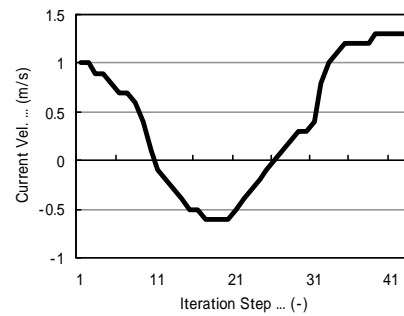


Fig. 6c Convergence of current velocity U (SUTP0000; 1 layer)

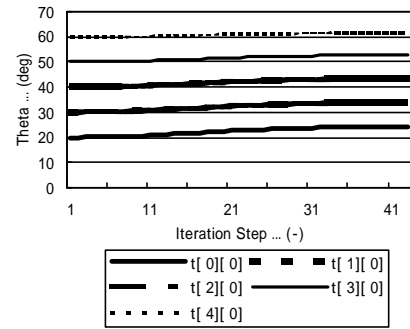


Fig. 6d Convergence of ray inclination angles q (SUTP0000; 1 layer; $t[m][n]$ denotes q_{mn})

The results in case of two layers where $h_0 = 300m$ and $h_1 = 700m$ are summarized in Table 5. In this calculation, $U_0 = U_1$ is assumed for the stability of convergence of the iteration calculation. Now, the error of the horizontal position of the bottom transponder is $-0.005m$. The acoustic and current velocities seem reasonable.

Table 5 The sound velocity, the current velocity, the rays and the horizontal position of the bottom transponder (SUTP0000; 2 layer)

	Acoustic Vel. C		Current Vel. U (m/s)	Pos. of B. T. x_B (m)
	C_0 (m/s)	C_1 (m/s)		
Average of Exact Sol.	1474.3	1472.9	1.4	0
2 Lyer Sol.	1484.8	1468.4	1.4	-0.005

5. CONCLUSION

In the previous reports, the current velocity was assumed zero. In the present report, the theory to include the effects of the current was constructed, and some two-dimensional numerical calculations were conducted. According to the results, the effects of the current can't be neglected to realize a high precision positioning of centimeter order necessary to the measurements of the sea bottom crust movements.

To introduce the effects of the currents fully, many studies must be done. When the acoustic and current velocities are unknowns, the most important problem is how to solve the difficulty in convergence of iterations as the number of the layers increases. The results will be reported in the coming reports.

On the other hand, an application of the present theory to a field other than the measurements of the sea bottom crust movements may also be pursued.

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