§8.6　修正された Hamilton-Dirichlet の原理 2

(8.4.4)式で与えられる Hamilton-Dirichlet の原理において, 水と板との連続条件(8.4.5b)式を拘束して， w を消去すると

$$\Pi_{HD}^{*2}[\phi]=\Pi_{KD}[w,\phi]_{w=\frac{1}{i\omega}\phi_z(x,y,0)}$$

$$=\frac{1}{2}\iint_R m{\phi_z}^2 dxdy$$

$$-\frac{D}{2\omega^2}\iint_R\left[\left(\frac{\partial^2\phi_z}{\partial x^2}\right)^2+2\nu\left(\frac{\partial^2\phi_z}{\partial x^2}\right)\left(\frac{\partial^2\phi_z}{\partial y^2}\right)+\left(\frac{\partial^2\phi_z}{\partial y^2}\right)^2+2(1-\nu)\left(\frac{\partial^2\phi_z}{\partial x\partial y}\right)^2\right]dxdy$$

$$-\frac{\rho g}{2\omega^2}\iint_R{\phi_z}^2 dxdy$$

$$-\frac{\rho}{2}\iiint_\Omega\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2\right]dxdydz$$

$$+\rho\iint_R\phi\phi_z dxdy$$

$$+\frac{\rho\omega^2}{2g}\iint_{S_F}\phi^2 dxdy$$

$$+\rho\iint_{S_O}\bar{f}_O\phi\, dS$$

$$=stationary$$

$$under\quad [K_{plate}^{*2}]+[M_{water}]$$

（8.6.1)

を得る。ただし，$[M_{water}]$は直接表に現れないので，省略してもよい。この場合の$[K_{plate}^{*2}]$は

$$M_x=-\frac{D}{i\omega}\left(\frac{\partial^2\phi_z}{\partial x^2}+\nu\frac{\partial^2\phi_z}{\partial y^2}\right)\quad in\ R \tag{8.6.2a}$$

$$M_y=-\frac{D}{i\omega}\left(\frac{\partial^2\phi_z}{\partial y^2}+\nu\frac{\partial^2\phi_z}{\partial x^2}\right)\quad in\ R \tag{8.6.2b}$$

$$M_{xy}=-(1-\nu)\frac{D}{i\omega}\frac{\partial^2\phi_z}{\partial x\partial y}\quad in\ R \tag{8.6.2c}$$

である。

実際に，$\delta\Pi_{HD}^{*2}=0$ を求めると

$$
\begin{aligned}
0 &= \delta\Pi_{HD}^{*2} \\
&= \delta\left[\begin{array}{l}
\dfrac{1}{2}\iint_R m{\phi_z}^2 dxdy \\
-\dfrac{D}{2\omega^2}\iint_R \left[\begin{array}{l}\left(\dfrac{\partial^2\phi_z}{\partial x^2}\right)^2+2\nu\left(\dfrac{\partial^2\phi_z}{\partial x^2}\right)\left(\dfrac{\partial^2\phi_z}{\partial y^2}\right)+\left(\dfrac{\partial^2\phi_z}{\partial y^2}\right)^2 \\ +2(1-\nu)\left(\dfrac{\partial^2\phi_z}{\partial x\partial y}\right)^2\end{array}\right]dxdy \\
-\dfrac{\rho g}{2\omega^2}\iint_R {\phi_z}^2 dxdy
\end{array}\right] \\
&\quad -\rho\iiint_\Omega\left(\frac{\partial\phi}{\partial x}\frac{\partial\delta\phi}{\partial x}+\frac{\partial\phi}{\partial y}\frac{\partial\delta\phi}{\partial y}+\frac{\partial\phi}{\partial z}\frac{\partial\delta\phi}{\partial z}\right)dxdydz \\
&\quad +\rho\iint_R(\delta\phi\,\phi_z+\phi\,\delta\phi_z)\,dxdy \\
&\quad +\frac{\rho\omega^2}{g}\iint_{S_F}\phi\,\delta\phi\,dxdy \\
&\quad +\rho\iint_{S_O}\bar{f}_O\delta\phi\,dS
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\omega^2}\iint_R\left[\begin{array}{l}i\omega\left(\dfrac{\partial^2 M_x}{\partial x^2}+2\dfrac{\partial^2 M_{xy}}{\partial x\partial y}+\dfrac{\partial^2 M_y}{\partial y^2}\right) \\ +m\omega^2\phi_z+\rho\omega^2\phi-\rho g\phi_z\end{array}\right]\delta w\,dxdy \\
&\quad -\frac{1}{\omega^2}\sum_{i=1}^{N}\int_{\Gamma_i}\left[-i\omega M_n\frac{\partial\delta\phi_z}{\partial n}+i\omega\left(Q_n+\frac{\partial M_{ns}}{\partial s}\right)\delta\phi_z\right]ds \\
&\quad +\frac{1}{\omega^2}\sum_{i=1}^{N}\left[i\omega M_{ns}\delta\phi_z\right]_{P_{i+}}-\frac{1}{\omega^2}\sum_{i=1}^{N}\left[i\omega M_{ns}\delta w\right]_{P_{i-}} \\
&\quad +\rho\iiint_\Omega\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial x^2}\right)\delta\phi\,dxdydz \\
&\quad -\rho\iint_R\left(\frac{\partial\phi}{\partial z}-\frac{\omega^2}{g}\phi\right)\delta\phi\,dxdy
\end{aligned}
$$

$$-\rho\iint_{S_O}\frac{\partial\phi}{\partial n}\delta\phi\,dS$$

$$-\rho\iint_{S_O}\left(\frac{\partial\phi}{\partial n}-\bar{f}_O\right)\delta\phi\,dS \qquad (8.6.3)$$

となる。したがって，この変分問題の自然条件は

$$i\omega\left(\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}\right)+m\omega^2\phi_z+\rho\omega^2\phi-\rho g\phi_z=0 \quad in\ R \qquad (8.6.4a)$$

$$i\omega M_n=0 \quad on\ \Gamma_{D_i}, \quad (i=1,2,\cdots,N) \qquad (8.6.4b)$$

$$i\omega\left(Q_n+\frac{\partial M_{ns}}{\partial s}\right)=0\ on\ \Gamma_{D_i}, \quad (i=1,2,\cdots,N) \qquad (8.6.4c)$$

$$i\omega M_{ns}=0 \quad at\ P_{i\pm}, \quad (i=1,2,\cdots,N) \qquad (8.6.4d)$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \quad in\ \Omega \qquad (8.6.4e)$$

$$\frac{\partial\phi}{\partial z}=\frac{\omega^2}{g}\phi \quad on\ S_F \qquad (8.6.4f)$$

$$\frac{\partial\phi}{\partial n}=0 \quad on\ S_B \qquad (8.6.4g)$$

$$\frac{\partial\phi}{\partial n}=\bar{f}_O \quad on\ S_O \qquad (8.6.4h)$$

で与えられる。

(8.6.2)式を代入すると，(8.6.4a)式は

$$D\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2\phi_z-m\omega^2\phi_z-\rho\omega^2\phi+\rho g\phi_z=0 \quad in\ R \qquad (8.6.5)$$

と書ける[8,9]。

§8.7 参考文献

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